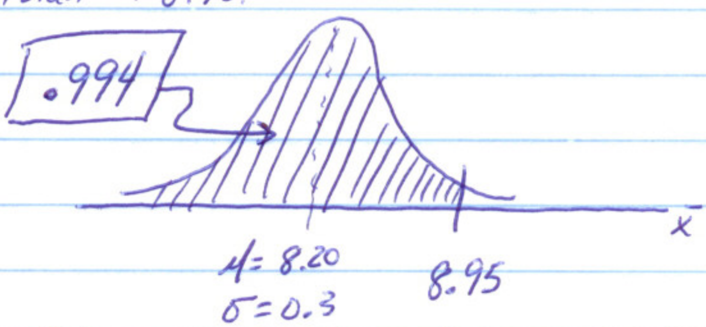
Math 106 Exam 2 Solutions
Spring 2009 - Hartlaub

1. $\underline{X}$ = expenses per customer

$\mu_x = \$8.20, \quad \sigma_x = \$3.00$

a. The mean of the sample average is $\mu_{\bar{x}} = \$8.20$ and the std error is $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{\$3.00}{\sqrt{100}} = \$0.30.$

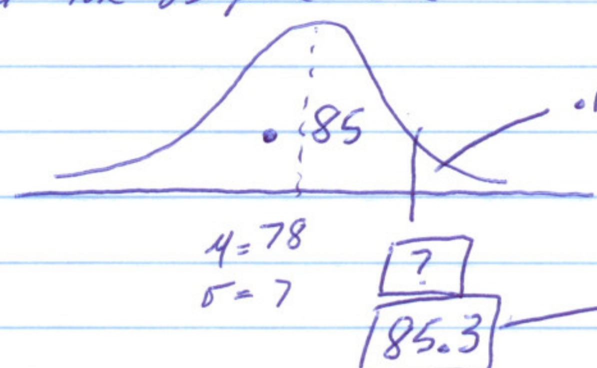
b. The company makes a profit when the sample mean is less than \$8.95.



Calc > Prob Dist > Normal (cumulative) .99379

2. X = score on midterm; $X \sim N(\mu_x = 78, \sigma_x = 7)$

Find the 85 percentile.



Graph > Prob Dist Plot > View Prob > right tail .15

OR Calc > Prob dist > Normal invcdf .85 $\Rightarrow \boxed{85.255}$

Yes, since $89 > 85.3$, you received an A.

Math 106 Exam 2 Solutions
Spring 2009 - Hartlaub

3. a. Let μ = the true mean commuting distance. We are given $\bar{x} = 10.9$, $n = 300$, and $s = 6.2$. Thus, a 99% CI for μ is (n is large & this is a R.S. so we can use the t interval)

$$\bar{x} \pm t^* \frac{s}{\sqrt{n}} \Rightarrow 10.9 \pm (2.58)\frac{6.2}{\sqrt{300}}$$

$$\Rightarrow 10.9 \pm .924$$

$$\Rightarrow (9.976, 11.824)$$

Minitab gives: (9.972, 11.828)

We are 99% confident that μ is contained by the interval (9.972, 11.828)

b. Yes, the interval in part (a) can be used to conduct an $\alpha = .01$ level test of $H_0: \mu = 12$ vs $H_1: \mu \neq 12$. Since 12 is not in the 99% CI for μ, we would reject the null hypothesis and conclude that the true mean commuting distance does not equal 12.

#4. a. Let π = the true proportion of apartments which prohibit children

$H_0: \pi = .75$

$H_1: \pi > .75$

Since $n\pi = 125(.75) = 93.75 > 10$, and $n(1-\pi) = 125(.25) = 31.25 > 10$, the large sample z test may be used.

$$z = \frac{p - .75}{\sqrt{\frac{.75(.25)}{125}}} = \frac{.816 - .75}{.0387} = 1.71$$

Math 106 Exam 2 Solutions
Spring 2009 - Hartlaub.

#4 a (cont) Minitab (Stat > Basic Stats > 1 Prop) [Events: 102, Trials: 125] gives

$z = 1.70$ and P-value = .044

Since .044 < .05, we reject H_0 and conclude that more than 75% of the apartments exclude children.

b. The $\alpha = .05$ level tests rejects H_0 if

$$\frac{\hat{p} - .75}{.0387} > 1.645 \implies \hat{p} > .75 + .0387(1.645) = .8137$$

$$\text{Power} = P(\hat{p} > .8137) = P\left(Z > \frac{.8137 - .8}{\sqrt{\frac{.8(.2)}{125}}}\right)$$

$$= P(Z > .3829) = .351$$

Stat > Power and Sample size > 1 Proportion

Sample size: 125
Alternative: .8
Hypothesized p: .75

Gives $\boxed{.350836}$

#5 Let μ_d = the true average difference in number of seeds detected by the two methods (Direct - Stratified)

$H_0: \mu_d = 0$ $\quad \alpha = .05$
vs $H_1: \mu_d \neq 0$

Normal prob plot of differences is NOT very linear - many students will rely on "large" sample size.

Test statistic:

$$t = \frac{\bar{x}_d}{s_d/\sqrt{n}} = \frac{-3.407}{13.253/\sqrt{27}} = -1.34 \text{ (with 26 d.f.)}$$

Math 106 Exam #2 Solutions
Brad Hartlaub - Spring 2009

#5 (cont) P-value = .193

Since .193 > .05 the null hypothesis is not rejected. The data do not provide sufficient evidence to conclude that the mean # of seeds detected differs for the two methods.

* Note: the same conclusion is made with nonparametric procedures (sign test or signed rank test).

#6. Let μ_1 = the true mean hardness for chicken chilled 0 hours before cooking and μ_2 = the true mean hardness for chicken chilled 8 hours before cooking

$n_1 = 36$, $\bar{x}_1 = 7.52$, $s_1 = 0.96$; $n_2 = 36$, $\bar{x}_2 = 5.70$, $s_2 = 1.32$

$H_0: \mu_1 = \mu_2$ vs $H_1: \mu_1 > \mu_2$ (Both sample sizes are > 30, so we rely on the CLThm & use the 2 sample t procedure)

Test stat:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = 6.69 \quad (\text{with} \simeq 63 \text{ d.f.})$$

P-value < .0001

Since the P-value < .05, we reject H_0 and conclude that the mean hardness is significantly higher for chicken chilled 0 hours before cooking than for chicken chilled 8 hours before cooking.

Math 106 Exam #2 Solutions
Spring 2009 - Hartlaub

#7. a. $\hat{p}$ is the midpoint of the CI, so

$$\hat{p} = \frac{.1788 + .3212}{2} = .25$$

(Taken from Fall 2008 Exam)

b. $$\text{Margin of error} = K\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$90\% \text{ CI} \Rightarrow K = 1.6449$$

$$2 \times \text{Margin of error} = .3212 - .1788 = .1424$$

$$\Rightarrow \text{Margin of error} = .0712$$

$$1.6449\sqrt{\frac{.25(.75)}{n}} = .0712$$

$$\Leftrightarrow \sqrt{\frac{.25(.75)}{n}} = \frac{.0712}{1.6449}$$

$$\Leftrightarrow n = \frac{.25(.75)}{.0019}$$

$$\Leftrightarrow n = 100.0738$$

There were 100 observations in the sample.